Applied Econometrics

1 Augmented Dickey Fuller, ADF, Test

Consider a simple general $AR(p)$ process given by

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + ...\phi_p Y_{t-p} + \epsilon_t \tag{1}$$

If this is the process generating the data but an $AR(1)$ model is fitted, say

$$Y_t = \mu + \phi_1 Y_{t-1} + \nu_t \tag{2}$$

then

$$\nu_t = \phi_2 Y_{t-2} + ...\phi_p Y_{t-p} + \epsilon_t \tag{3}$$

and the autocorrelations of v_t and ν_{t-k} for $k > 1$, will be nonzero, because of the presence of the lagged Y terms. Thus an indication of wheter it is appropriate to fit an $AR(1)$ model can be aided by considering the autocorrelations of the residual from the fitted models.

To illustrate how the DF test can be extended to autoregressive processes of order greater than 1, consider the simple $AR(2)$ process below.

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t \tag{4}$$

then notice that this is the same as:

$$Y_t = \mu + (\phi_1 + \phi_2) Y_{t-1} - \phi_2 (Y_{t-1} - Y_{t-2}) + \epsilon_t \tag{5}$$

and subtracting Y_{t-1} from both sides gives:

$$\Delta Y_t = \mu + \beta Y_{t-1} - \alpha_1 \Delta Y_{t-1} + \epsilon_t \tag{6}$$

where the following have been defined

$$\beta = \phi_1 + \phi_2 - 1$$

and

$$\alpha_1 = -\phi_2$$

Figure 1: Graph of Levels and Correlogram of Y variable

This means the if the appropriate order of the AR process is 2 rather than 1, the term $\Delta Y_{t-1} should$ be added to the regression model. A test of wheter there is a unit root can be carried out in the same way as for the DF test, with the test statistics provided by the $'t'$ statistics of the β coefficient. If $\beta = 0$ than there is a unit root. The same reasoning can be extended for a generic $AR(p)$ process. Therefore to perform an Uniti Root test on a $AR(p)$ model the following regression should be estimated:

$$\Delta Y_t = \mu + \beta Y_{t-1} - \sum_{j=1}^{p} \alpha_j \Delta Y_{t-j} + \epsilon_t \tag{7}$$

The Standard Dickey-Fuller model has been 'augmented'by ΔY_{t-j}. In this case the regression model and the $'t'$ test are referred as the ADF test.

1.0.1 Practical Implementation of ADF test in PcGive

Let's see how you implement the ADF test in PcGive. Consider the generic variable Y graph in Figure 1.

This variable Y is clearly trended and you have to determine if this trend is stochastic or deterministic. After having created the difference variable ΔY, estimate the following model, with as many lag of ΔY as you think appropriate.(in the example I choose 4 lags of the variable ΔY)

$$\Delta Y_t = \mu + bt + \beta Y_{t-1} - \alpha_1 \Delta Y_{t-1} + \alpha_2 \Delta Y_{t-2} + \alpha_3 \Delta Y_{t-3} + \alpha_3 \Delta Y_{t-4} + \epsilon_t \tag{8}$$

Figure 2: Set Up The Model

Estimate the equation by OLS, and obtain the resultsshown in Figure 3 (you can obtain the results as shown above selecting PcGive- Model - Options Menu and selecting the options shown in Figure 4).

The results show the absence of serial correlation in the residual. This means that you can start reducing the model, estimating sequencially a more parsimonious model, like:

$$\Delta Y_t = \mu + bt + \beta Y_{t-1} - \alpha_1 \Delta Y_{t-1} + \alpha_2 \Delta Y_{t-2} + \alpha_3 \Delta Y_{t-3} + \epsilon_t \tag{9}$$

$$\Delta Y_t = \mu + bt + \beta Y_{t-1} - \alpha_1 \Delta Y_{t-1} + \alpha_2 \Delta Y_{t-2} + \epsilon_t \tag{10}$$

$$\Delta Y_t = \mu + bt + \beta Y_{t-1} - \alpha_1 \Delta Y_{t-1} + \epsilon_t \tag{11}$$

$$\Delta Y_t = \mu + bt + \beta Y_{t-1} + \epsilon_t \tag{12}$$

In analysing the regression output, you should look at the t values for the lag of ΔY , the autocorrelation and normality tests, or perform specific F-test

Figure 3: PcGive Results

Figure 4: Model-Options Menu

Figure 5: Model-Progress Results

(in the same way you have already studied in the first part of the course). An usefull utility you have in PcGive is the Progress analysis, which compare the model specifications (8) to (12). After having perform the five OLS regression in sequence, select Model-Progress and obtain the results shown in Figure 5. Both the F-Tests and the Schwatz Information Criteria indicates that the model with one lag in the difference (number 4 of the results) is the one to be preferred as base for the unit root analysis.

1.1 Testing For Unit Root

Having selected the model following a standard model reduction procedures, we have now to enter the proper unit root analysis. Because the data are trended, the purpose of the Unit Root test is to determine whether the series is consistent with an $I(1)$ process with a stochastic trend, or if it is consistent with an $I(0)$ process, that is it is stationary, with a deterministic trend. Given the model selected above, the hypotesis can be formally formulated as:

$$\Delta Y_t = \mu + bt + \beta Y_{t-1} - \alpha_1 \Delta Y_{t-1} + \epsilon_t \tag{13}$$

Figure 6:

$$H_0 : (\mu, b, \beta) = (\mu, 0, 0) \cdots v \cdots H_1 : (\mu, b, \beta) \neq (\mu, 0, 0)$$

Therefore an F-test is performed on the join hypotesis $b = \beta = 0$ and the results should be compared with the ϕ_3 test. Pratically, after having estimated the equation above, select PcGive Menu - Test - Linear Restriction Subset, and you will have the menu shown in Figure (6). After selecting the variable of interest (*trend* and Y_1), the results are shown in Figure (7)

Wald test for linear restrictions: Subset
LinRes F(2,490) = 4.9655 [0.0073] **

According to the values of the ϕ_3 test, we cannot reject the hypotesis of random walk without drift. This means that a deterministic trend is not present in our series.

The next step is therefore is to use the t statisitcs to test for $\beta = 0$ given the fact that the time trend hypotesis has been rejected. Estimating the model

$$\Delta Y_t = \mu + \beta Y_{t-1} - \alpha_1 \Delta Y_{t-1} + \epsilon_t \tag{14}$$

we obtain the results in Figure (8). The $t - test$ of -0.499 (compare with a limit value of -2.89 at 95% confidence) seems to confirm that the series has unit root.

The final step is to check if the drift is present in the series. Using PcGive menu - Test - Linear Restriction menu as before, we test if:

Figure 7:

$$H_0 : (\mu, \beta) = (0, 0) \cdots v \cdots H_1 : (\mu, \beta) \neq (0, 0)$$

Because we know that the series contain Unit Root, if the H_0 Hypotesis is rejected, we know that the series should contain a drift term. The results of the test is

Wald test for linear restrictions: Subset
LinRes F(2,491) = 43.445 [0.0000] **
Confronting this result with ϕ_1 confidence interval of 4.71, we cannot reject the hypothesis that the series contain a drift term. Therefore the final estimated identify the series as an $AR(2)$ model with unit root.